The *invisible wall* project: problem solving processes of pupils who work on problems with a goal which can not be reached

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The *invisible wall project* analyses problem solving processes of children in grades 3-4 and 8-9. By now, we have transcribed more than 160 interviews; as a consequence, the qualitative interpretive methods were complemented by quantitative methods. Besides analysis of *difficulty of tasks* and *learning while being tested*, we describe the *process of problem solving* by *action profiles*. Using the method of *divergent coding*, we introduce interpretive methods to better understand action profiles.

The *invisible wall* project

Since 1992 M. Stein has been working - together with several groups of student teachers - on problem solving. The focus of the research is on such components of problem solving ability which are *not* subject of mathematics lessons. As a consequence, the project does not deal with word problems and other Aclassical problems@. The central idea of the research is to use sets of tasks which are all *unsolvable* which means they have a goal which can not be reached. The unsolvability, however, is of a kind which can be understood even by younger children (e.g.: try to find exactly 4 different numbers out of the set {1, 2, 3, 4, 5} which give the sum 9). Subsequently, we shall use the term *impossible task* as well. (See STEIN 1998, this volume, for more informations)

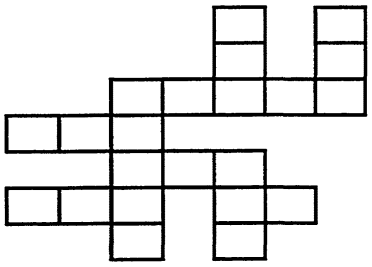
In the first part of the project elementary components of problem solving behaviour which are actually used by younger children were identified (the methods and some of the results are described in STEIN 1997). The search for those components was organized as search for noticeable patterns in the subject=s behaviour. The impossibility of a solution acts as an **Ainvisible wall@** for the pupils' actions: the pupil tries to solve the task, gets stuck ("bounces against the invisible wall"), and tries a different solution.

In the present stage of the project the authors try to describe profiles of problem solving processes. Funded with a grant from *Deutsche Forschungsgemeinschaft*, we have protocolled and fully transcribed app. 160 interviews with pupils of grades 3-4 and grades 7-8. This material is now open to analysis by *quantitative* methods.

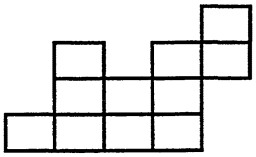
The tasks

We work with a range of different puzzles. In this paper, we concentrate on the following two puzzles:

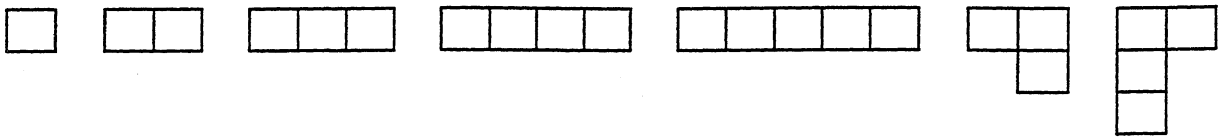
Puzzle 1 „big puzzle“



Puzzle 2 „small puzzle“

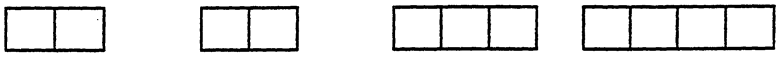


The big puzzle shall be filled exactly with the following parts (each is given once).



unit-bar | 2-unit-bar | 3-unit-bar | 4-unit-bar | 5-unit-bar | 2-unit-angle | 3-unit-angle

For the small puzzle, there are the following parts:



2-unit-bar | 2-unit-bar | 3-unit-bar | 4-unit-bar

Design of the study

All problems are solved by groups of two pupils. The pupils are informed that some of the tasks can be solved, other tasks not, and that they have to find out what is the case and why.

As has been said, the search for components of problem solving behaviour is a search for noticeable patterns in the actions of the children. We do *not* assume, however, that such a pattern is guided by a consciously applied "strategical insight". If a teacher - or, in our case, the interviewer - starts asking questions about the behaviour, the pupils may discover the strategical implications of their behaviour and start talking about it as if they had planned their proceedings from the very beginning.

As a consequence, the interviewers *only watch the pupils work*. When the pupils *say* that the task can not be solved, or *show some "suspicion"* the interviewers ask whether there might be other ways to solve the task. This may be repeated once. Only at the end of the interview the interviewers are permitted to talk with the children about their understanding of the situation and about the process of solution.

The interviews are filmed with a video camera. Every action of the children is protocolled. The final transcription has the character of a script for a movie which allows to replay the interview.

The script is interpreted turn by turn (MAIER 1991) by a team of interpreters which includes the interviewer. The behavior of the children is analysed under a broad range of aspects. In many cases there will be more than one interpretation of the same behaviour. The same action may be interpreted, for instance, as a consequence of a social conflict between the two children, or may be seen as influenced by gestalt operations or be understood as guided by explicit use of heuristic strategies.

The tasks were presented to pupils in grades 3/4 and to pupils in grades 7/8. In grades 3/4 we tested 37 pairs of pupils of all abilites, from 8 different schools. In grades 7/8 we tested 46 pairs of pupils of all abilites, from 7 different schools.

Each pair of pupils got 4 tasks, the first two of them always were solvable. After that, each pair of pupils got one big and one small puzzle. The order (big puzzle first, or big puzzle as second task) was randomly assigned to the pairs of pupils.

Categorization of the final argumentation

At the end of each interview the pupils are encouraged to give reasons for their opinion that the task can not be solved. It may be assumed that the answers are an indicator for the pupils' understanding of the unsolvability of the task. We have developed categories for grading the tasks

by the completeness and exactness of the answers. By the results, we can get informations about the difficulty of the tasks.

The answers are classified using three different categories:

Category I Systematic Reasoning or mention of the relevant fact.
It is said that the two big pieces are essential for the unsolvability of the task.
Examples: "The 5-unit-bar and the 4-unit-bar have fixed positions on the puzzle. But, if they are laid down, I have no place for the 3-unit-angle." - „It can not be done because the 5-unit-bar can be laid down in two different positions only."

Category II Modification of basic assumptions
The pupil makes comments about necessary modifications of the rules or the pieces of the puzzle "to make it solvable".
Example: "If I could cut the 4-unit-bar into two pieces, I could solve the task."

Category III Mention of isolated facts
The pupils mentions some facts which he observed as obstacles for the solution of the task.
Example: "There is always one piece left."

The classification of the pupils' answers by those categories was done by one of the authors. A choice of tasks and the coding rules were given to two independent coders. The results of the three coders were analysed using the Q-concordance-test by Gebert and Lienert (1971).

Difficulty of tasks

We assume that pupils have understood the unsolvability of the task if they are able to explain systematically why the task can not be solved (Category I). Pupils who gave answers in categories II or III *may* have understood why the puzzle is not solvable, but they seem not to be able to put their insight in good words. So, we take the numbers of answers in category I as an indicator for the difficulty of the tasks.
We ask:
- are "small" puzzles easier than "big" puzzles?
- do pupils in grades 3/4 have more difficulties with the tasks than pupils in grades 7 to 9?

The zero hypothesis is always, that there is no difference. The significance level is 5%.

We get the following results:

small puzzles

	grades 3/4	grades 7/8
I	23	30
II / III	14	16

Table 1

big puzzles

grades ¾	grades 7/8
15	28
22	18

Table 2

There are no significant differences between younger and older pupils in their respective ability to talk about the unsolvability the tasks (Altogether, even the older pupils do not show a

convincing performance).

Learning in the test situation

While working on a task, the pupil considers different aspects of it. Though he/she may not be conscious of it, his/her considerations may lead to learning effects.
As was said in the first section, every pair of pupils got two unsolvable tasks, a small puzzle and a big one. The presentations

- first small puzzle, then big one
- first big puzzle, then small one

were (nearly) equally distributed.

We now analyse whether the second unsolvable puzzle was solved better than the first unsolvable puzzle.

In grades 3/4 we find the following situation:

	first task big puzzle	second task small puzzle
I	7	10
II / III	9	6

Table 3

first task small puzzle	second task big puzzle
13	8
8	13

Table 4

In grades 7/8 we find the following situation:

	first task big puzzle	second task small puzzle
I	14	20
II / III	11	5

Table 5

first task small puzzle	second task big puzzle
10	13
11	8

Table 6

Tables 3, 4 and 6 show no significant differences between the groups. Table 5 shows a high increase in quality from first task to secons task. However, it does not pass the χ^2 - test for significance. An analysis using the exact test of Fisher shows, however, that the probability to get by chance this or an even more extreme result, is not much higher than 5%.

Since table 3 shows an increase in quality from first task to second task, we can get some additional information by putting both tables together (though this was not part of the original hypothesis).

	First task big puzzle	2nd task small puzzle
I	21	30
II/III	20	11

The increase in N leads to a table in which the increase in quality from 1st task to 2nd task is significant on the 5% - level.

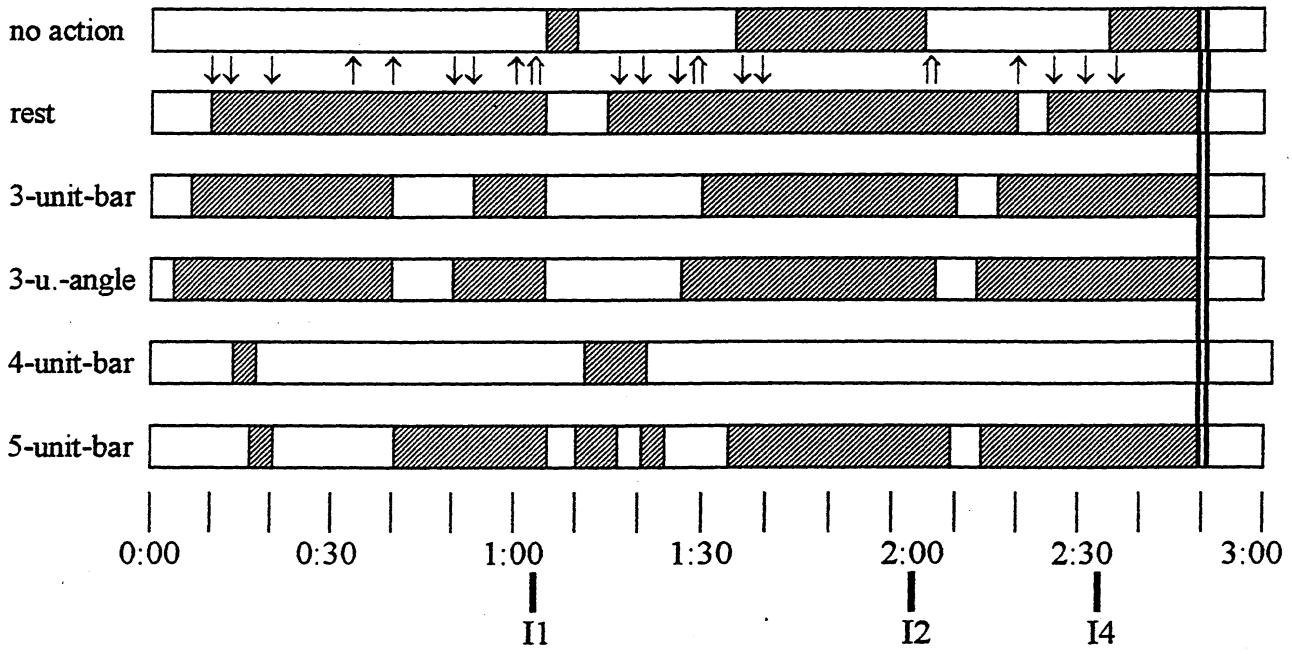
Action profiles and divergent coding

Looking at the results above we have to take into account that we do not know whether a pupil who gave an answer in category III *really* did not understand the unsolvability of the task. It

is possible that during his/her attempts in solving the task he/she acquired a good insight in the unsolvability, but found it so "self-evident" that he/she saw no necessity for giving a systematic argument. If we have pupils who gave "good" explanations in Category I, we are interested to find out where the insight in the reasons for the impossibility of the task happened.

To get more and better information about the connections between problem solving processes and the final "outspoken" answer, we analyse the action profiles of the problem solving processes by divergent coding (see STEIN 1998, this volume). Within the limits of this report, we can discuss three examples.

Example I

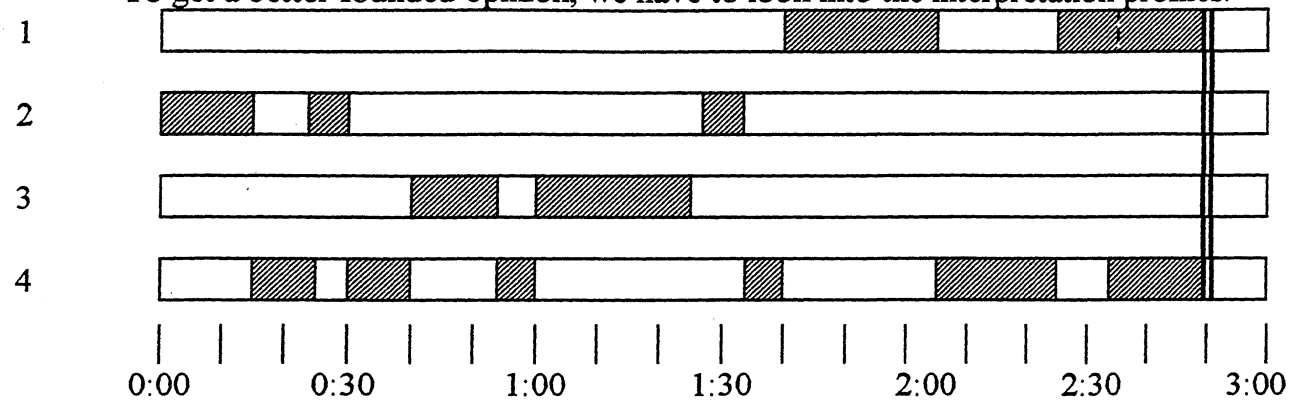


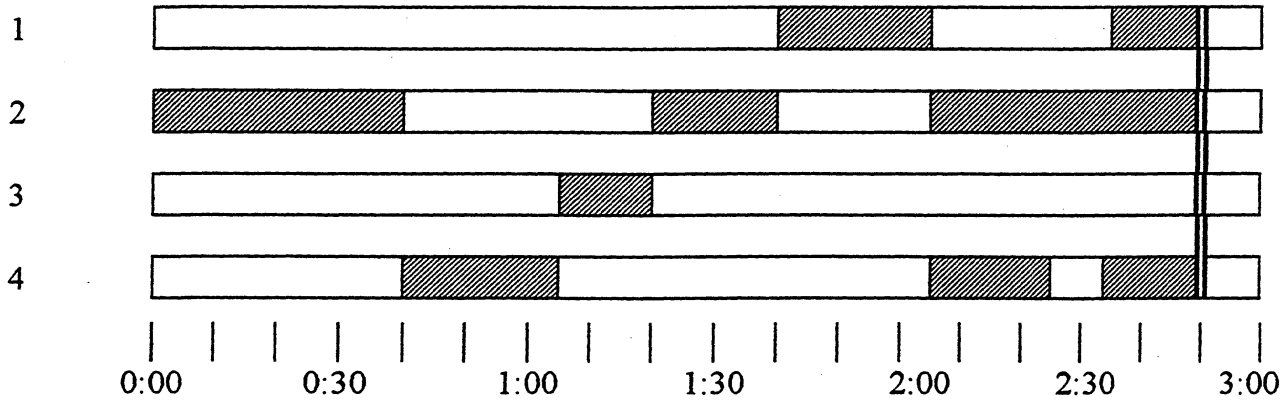
Two boys, both 13 years old and of average mathematical abilities, end up with the following pieces lying on the puzzle: 5-unit-bar, 3-unit-bar, 3-unit-angle. They say that the puzzle can not be solved : "The 4-unit-bar must be cut into two pieces, because there are not enough 2-unit-bars". This answer is categorized as Category II.

The action profile of the boys shows that the 5-unit-bar was nearly all the time lying on the puzzle. Additionally, nearly all the time we find the 3-unit-angle and/or the 4-unit-bar lying on the puzzle.

The combination of 5-unit-bar/4-unit-bar resp. 5-unit-bar/3-unit-angle is essential for seeing that the task is impossible: the 5-unit-bar/4-unit-bar - combination makes it impossible to put the 3-unit-angle down; the combination 5-unit-bar/3-unit-angle makes it impossible to put the 4-unit-bar down. (We call those combinations *blockade situations*). We may conclude that the boys "had the reasons for the impossibility of the task in mind" but found them after nearly 3 minutes of acting so self-evident that they did forget to formulate them. On the other hand, we find many actions with the small pieces. By this we may conclude that - though the big pieces were lying on the puzzle nearly all the time, the pupils did *not* recognize that this fact was the essential reason for the impossiblity of the task.

To get a better founded opinion, we have to look into the interpretation profiles.





We see many differences in the interpretation profiles of both coders. There is not much unanimity whether actions belong to *gestalt operations* or to *other systematic behavior* (for instance, trying to fill up the puzzle, beginning at one end). This should not surprise us: following the *gestalt* of a figure can - with our puzzles - lead to a behavior which looks very systematical.

Both coders see, however, only a short period of *logically guided behavior* - coder 1 from 0:40 to 1:30, coder 2 from 1:05 to 1:20. We understand this better by having another glance on the action profile: only at 1:10, the two big pieces are laid down with no other pieces lying on the puzzle. *At all other times the 5-unit-bar resp. 3-unit-angle are laid down with some small pieces already on the puzzle.* Placing the big pieces on the puzzle seems to be part of a "filling - up - action", not part of logically guided sequence.

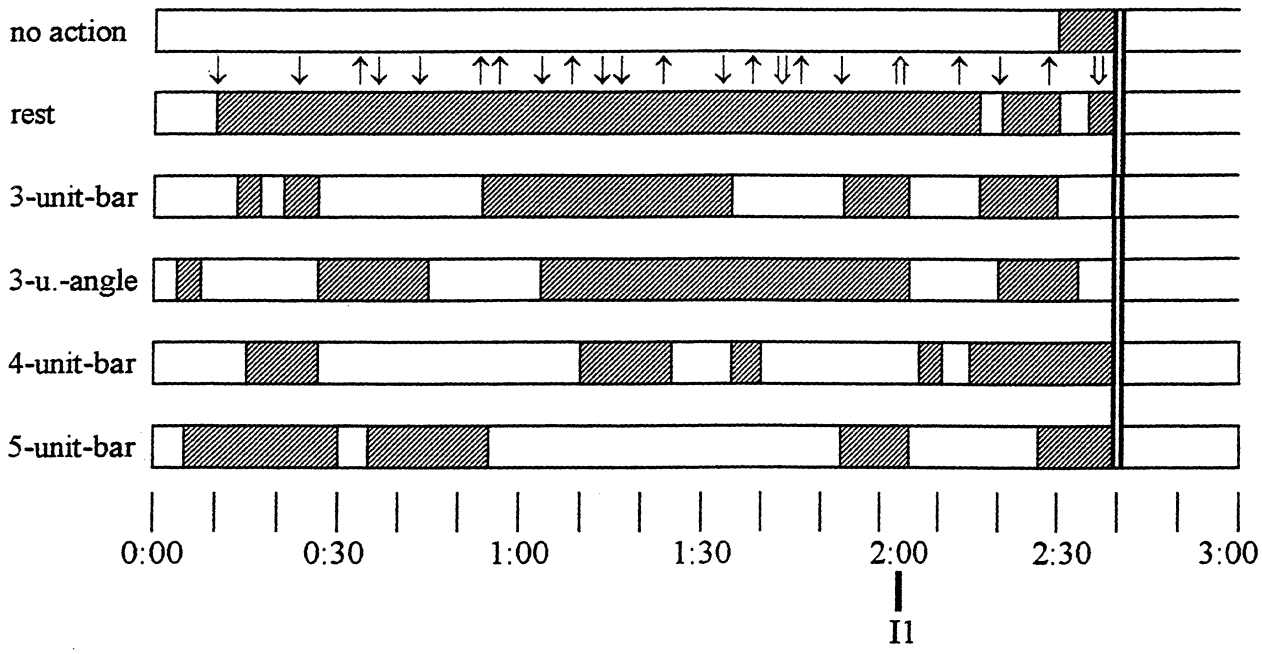
Conclusion In this case, we assume that the low quality of the outspoken answer reflects a problem solving process which did not use a basic analysis of the material. All attempts to fill the puzzle up ended in a situation in which the 4-unit-bar was left over or in which there were two free spaces with a 2-unit-length. This experience is summarized in the answer.

Example II

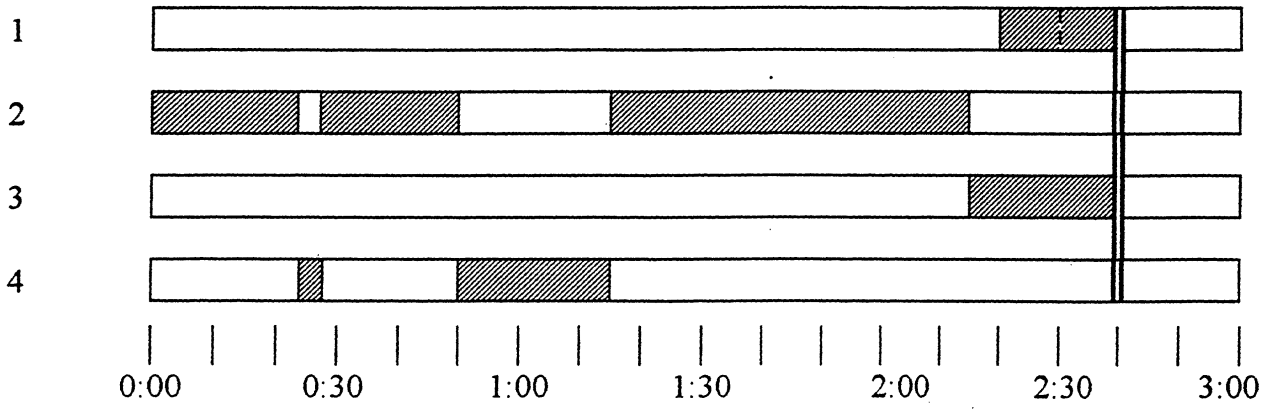
Two boys (12 y; 8m / 13 y; 5 m) of average mathematical ability, work for 2 minutes and 40 seconds. They end up with the 4-unit-bar and the 5-unit-bar in the right place. Their final argumentation is in Category I: "The 5-unit-bar and the 4-unit-bar have fixed positions on the puzzle. In any case there remain two free spaces of length 2, and there are not enough 2-unit-bars to fill those spaces up."

We now take a glance on the action profile. It shows many actions, involving pieces of all

sizes. During the first 2:20 - period, there are only very short periods containing blockade situations.



Since both interpretive coders nearly fully agreed in their interpretation, we give only one interpretation profile. It shows that until 2:20 the pupils follow *gestalt* operations or show *other systematic behavior*.



Since the argumentation of the pupils was so good, we shall try to find out where this insight happened.

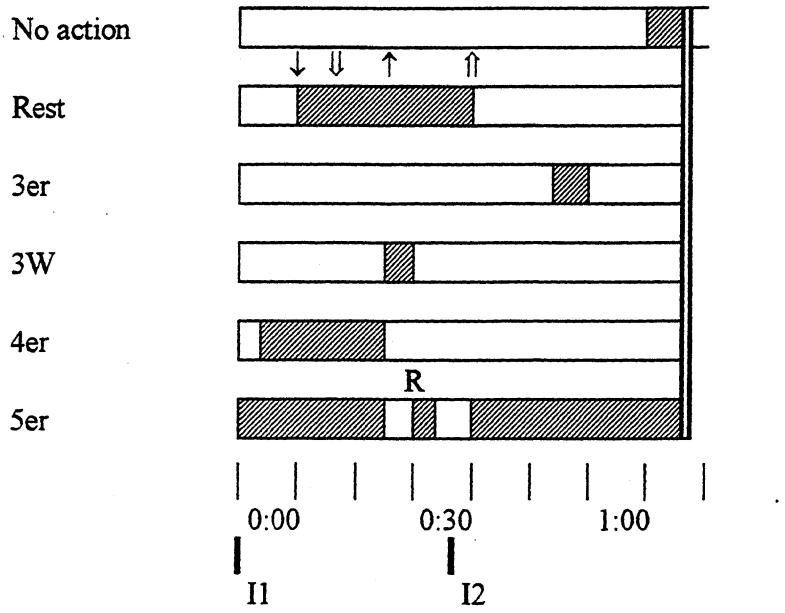
They started their work with the 5-unit-bar in it's place, and created some blockade positions. Since this was not successful, from 0:55 to 1:55 they worked without the 5-unit-bar. During this working phase they seem to have realised that the 5-unit-bar has only two possible positions, and has to be placed first. Some attempts follow to fill the puzzle up. Now the big pieces are always involved. At 2:35, the 3-unit-angle, the 4-unit-bar and the 5-unit-bar are lying on the puzzle. The 5-unit-bar is the last big piece laid down. The reader may convince himself that this *must* be against the rules.

Conclusion The end phase of the pupils' work seems to be the most interesting part of the interview. Having been unsuccessful for nearly two minutes, leads to increased number of actions involving big pieces. Those actions stabilize the insight in the reasons for the impossibility of the task. This insight is summarized in the answer.

Example III

Within the space of this paper, we can only summarize the results of the interpretive coding of the following transcript. The transcript shows the work of a 13 year old boy (medium ability level) and 13 year old girl (good in mathematics).

At the end of the interview, the pupils give the following explanation for the unsolvability of the task: „It cannot be done because the 5-unit-bar can not be laid down in another way".



We see that the pupils begin with the combination 5-unit-bar /4-unit-bar. This is a strong hint that they were guided by some logical considerations. After that, we see some actions with small pieces which may be seen as following the chosen logical path, or as gestalt reaction. From 0:30, the 5-unit-bar is lying on the puzzle all the time. We conclude that the pupils have realised, that this piece *must* be placed first. Since the pupils stop work until the interviewer starts talking to them (I2), we conclude that there answer - though very short - is a result of a clear insight *that* the task is unsolvable, and *why* it is unsolvable.

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